

M434 Differential geometry

Level: 3 Credit points: 30

No computer required

Summary

We develop the interactions between linear algebra and differential calculus to study the main ideas and examples of classical differential geometry, using modern mathematical techniques where they simplify or clarify the exposition. All the curves and surfaces studied are in three-dimensional Euclidean space and so can be readily visualized and described explicitly in terms of standard functions. The course is based on the set book Elementary Differential Geometry by B. O'Neill (Academic Press).

Description

The course develops the interaction between linear algebra and differential calculus in order to study the main ideas and examples of classical differential geometry, using modern mathematical techniques where they simplify or clarify the exposition. All the curves and surfaces studied are in three-dimensional Euclidean space and so can be readily visualized and explicitly described in terms of standard functions. The course is based on the set book, with a combined course guide and reminder of the mathematics you are expected to be already familiar with.

The six parts of the course cover the first six chapters of the set book. *Chapter I* extends the basic ideas of linear algebra and differential calculus to deal with vector fields, directional derivatives, differential forms and derivative mappings. An additional section deals with the chain rule.

Chapter II studies curves in terms of their tangent, normal and binormal vector fields. This Frenet frame leads to definitions of the curvature and torsion of a curve. Techniques for calculating these are developed and they are shown to reflect the geometry.

Chapter III studies mappings that preserve distance - isometries - and shows that if two curves have the same speed, curvature and torsion then there exists an isometry mapping one to the other.

Chapter IV shows how surfaces can be described in terms of co-ordinate patches. Typical surfaces are the sphere, the torus and the helicoid. Tangent vectors are constructed and used to describe calculus on a surface.

Chapter V uses the calculus to define the shape operator of a surface. This is a linear mapping on tangent vectors and can be represented by a 2×2 matrix. This matrix is used to define the Gaussian curvature (the determinant) and the mean curvature (the trace), and the principal directions (the eigenvectors). Formulae for calculating these are found and their relationship with the geometry of the surface and special curves (principal curves, asymptotic curves and geodesics) is discussed.

Chapter VI is about which properties of a curve are intrinsic, that is, which depend on the internal structure of the surface and not on how we describe them as subspaces of a larger space. We show that Gaussian curvature is such a property, and obtain an alternative method of calculating it.

Entry

The course assumes that you have already met certain basic mathematical concepts and techniques. Some of these are briefly reviewed in the course, but it is still advisable to have some familiarity with them before you begin. They are:

• Vector spaces, especially $\mathbb{R}^2$ and $\mathbb{R}^3$. Bases and co-ordinates.

• Dot (or scalar) product; lengths and angles. Orthogonality. Orthonormal bases, orthonormal

expansion.

• Linear transformations between vector spaces. Representation of a linear transformation by a matrix.

• Determinants. Calculation of determinants of 3×3 matrices.

• Cross (or vector) product.

• Definition of characteristic values and characteristic vectors (eigenvalues and eigenvectors) of a linear transformation (of a vector space to itself).

• Isometries: translations, rotations, reflections. Orthogonal matrices.

• Standard functions: polynomials, exp, log, cos, sin, tan, cosh, sinh. Properties of these functions and functions made up from them. Graphs of such functions. Differentiation of such functions.

• Trigonometric identities, especially $\cos^2 x + \sin^2 x = 1$. Corresponding identities for cosh and sinh: $\cosh^2 x - \sinh^2 x = 1$.

• Integration of simple functions.

• Partial differentiation. Calculation of partial derivatives of functions of two or three variables. (There is a brief review of these ideas in the course guide.)

You could get all this from the Level 2 courses M203 *Introduction to pure mathematics* and MST207 *Mathematical methods, models and modelling* (or the earlier course MST204 *Mathematical models and methods*). Your Regional Centre will be able to tell you where you can look at reference copies, or you can buy selected materials from Open University Worldwide Ltd.

This is a Level 3 course, which makes intellectual demands appropriate to the final year of an honours degree. If you have any doubt about your ability to take the course please seek advice from your Regional Centre..

Preparatory work

If you would like to acquaint yourself with some of the main elements of the course, the first chapter of the set book gives a good introduction.

If you are disabled

The course is not recommended if you have impaired sight, because diagrams in the set book are used in the assignments and recordings of printed course materials are not available at present. If you are a new student, make sure that you have our booklet *Meeting Your Needs*.

What's included

Course books, other printed materials.

Set book to buy

B. O'Neill *Elementary Differential Geometry*, (2nd edition), Academic Press, £34.95 (hardback)

Tuition and counselling

You will have a tutor who will help you with the course material and mark and comment on your written work, and whom you can ask for advice and guidance. We may also be able to offer group tutorials or day-schools that you are encouraged, but not obliged, to attend. Where your tutorials are held will depend on the distribution of students taking the course. Ask your Regional Centre if you need to know more before you decide whether to register. Your Regional Centre will provide a counselling service to give you general help with your studies.

Assessment

There are four tutor-marked assignments and an examination. Assessment is an essential part of the teaching, so you are expected to complete it all. But if you unavoidably miss or do badly in an assignment some courses allow you a 'substitution score', calculated as a weighted average of all your scores for the course. In M434 this rule can apply to one assignment. You will be given more detailed information when you begin the course.

Qualifications

This is a specified course in our MMath degree, and in our honours BSc in Mathematical Sciences and Computing, and in Mathematical Sciences. It can also count towards most of our other degrees at bachelor's level - it is equally appropriate to a BA or a BSc - and towards our discontinued Advanced Diploma in Mathematics Education. It is up to you to ensure that you are properly informed about the circumstances in which the course can count towards these qualifications.

Professional recognition

This course can help you to gain recognition from a professional body. Ask your Regional Centre for *Recognition* leaflets 3.3 and 3.6.

For the future

This course is presented every four years, and 2001 may be its last year.